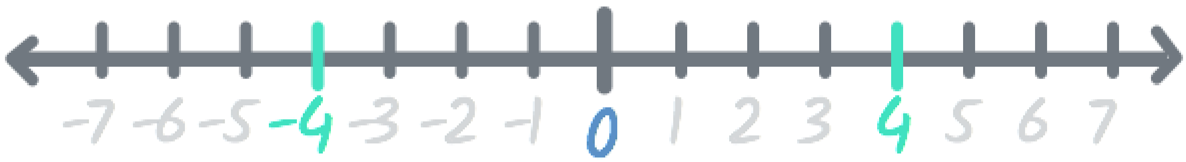
Lesson 4: Reciprocals and Inverse Numbers

Opposite numbers

Every number has an **opposite**. In fact, every number has **two** opposites: the **additive inverse** and the **reciprocal**—or **multiplicative inverse**. Don't be intimidated by these technical-sounding names, though. Finding a number's opposites is actually pretty straightforward.

The additive inverse

The first type of opposite is the one you might be most familiar with: **positive numbers** and **negative numbers**. For example, the opposite of **4** is **-4**, or **negative four**. On a number line, 4 and -4 are both the same distance from **0**, but they're on opposite sides.



This type of opposite is also called the **additive inverse**. **Inverse** is just another word for **opposite**, and **additive** refers to the fact that when you **add** these opposite numbers together, they always equal **0**.

$$-4 + 4 = 0$$

In this case, **-4 + 4** equals **0**. So does **-20 + 20** and **-*x* + *x***. In fact, any number you can come up with has an additive inverse. No matter how large or small a number is, adding it and its inverse will equal 0 every time.

If you've never worked with positive and negative numbers, you might want to review our lesson on **negative numbers**.

To find the additive inverse:

For positive numbers or variables, like 5 or *x*: Add a negative sign (-) to the left of the number: 5 → -5.

$$\begin{array}{ccc} x & \longrightarrow & -x \\ 3y & \longrightarrow & -3y \end{array}$$

For negative numbers or variables, like -5 or *-x*: Remove the negative sign: -10 → 10.

$$\begin{array}{ccc} -y & \longrightarrow & y \\ -6x & \longrightarrow & 6x \end{array}$$

Using the additive inverse

The main time you'll use the additive inverse in algebra is when you **cancel out** numbers in an expression. (If you're not familiar with cancelling out, check out our lesson on **simplifying expressions**.) When you cancel out a number, you're eliminating it from one side of an equation by performing an **inverse action** on that number on **both** sides of the equation. In this expression, we're cancelling out **-8** by adding its **opposite: 8**.

$$\begin{aligned} x - 8 &= 12 \\ +8 \quad & \quad +8 \end{aligned}$$

Using the additive inverse works for cancelling out because a number added to its inverse **always** equals **0**.

Reciprocals and the multiplicative inverse

The second type of opposite number has to do with **multiplication** and **division**. It's called the **multiplicative inverse**, but it's more commonly called a **reciprocal**.

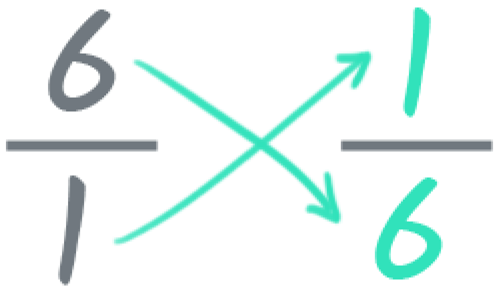
To understand the reciprocal, you must first understand that every whole number can be written as a **fraction** equal to that number divided by **1**. For example, **6** can also be written as **6/1**.

$$6 = \frac{6}{1}$$

Variables can be written this way too. For instance, **x = x/1**.

$$x = \frac{x}{1}$$

The **reciprocal** of a number is this fraction flipped upside down. In other words, the reciprocal has the original fraction's bottom number—or **denominator**—on top and the top number—or **numerator**—on the bottom. So the reciprocal of **6** is **1/6** because 6 = 6/1 and 1/6 is the **inverse** of 6/1.



Below, you can see more reciprocals. Notice that the reciprocal of a number that's already a fraction is just a flipped fraction.

$$5y \longrightarrow \frac{1}{5y}$$

$$18 \longrightarrow \frac{1}{18}$$

$$\frac{3}{4} \longrightarrow \frac{4}{3}$$

And because reciprocal means **opposite**, the reciprocal of a reciprocal fraction is a **whole number**.

$$\frac{1}{7} \longrightarrow 7$$

$$\frac{1}{2} \longrightarrow 2$$

$$\frac{1}{25} \longrightarrow 25$$

From looking at these tables, you might have already noticed a simpler way to determine the reciprocal of a whole number: Just write a fraction with **1** on **top** and the original number on the **bottom**.
Decimal numbers have reciprocals too! To find the reciprocal of a decimal number, change it to a fraction, then flip the fraction. Not sure how to convert a decimal number to a fraction? Check out our lesson on **converting percentages, decimals, and fractions**.

Using reciprocals

If you've ever **multiplied** and **divided fractions**, the reciprocal might seem familiar to you. (If not, you can always check out our lesson on **multiplying and dividing fractions**.) When you multiply two fractions, you multiply straight across. The numerators get multiplied, and the denominators get multiplied.

$$\frac{4}{5} \cdot \frac{2}{3} = \frac{8}{15}$$

However, when you **divide** by a fraction you flip the fraction over so the numerator is on the bottom and the denominator is on top. In other words, you use the **reciprocal**. You use the **opposite** number because multiplication and division are also opposites.

$$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \cdot \frac{3}{2} = \frac{12}{10}$$

Practice!

Use the skills you just learned to solve these problems. After you've solved both sets of problems, you can scroll down to view the answers.

Practice set 1

Find the **additive inverse**:

5 ,-8,q

Practice set 2

Find the **reciprocal**:

5,5/6,0.75